Scrambling Dynamics with Imperfections in A Solvable Model

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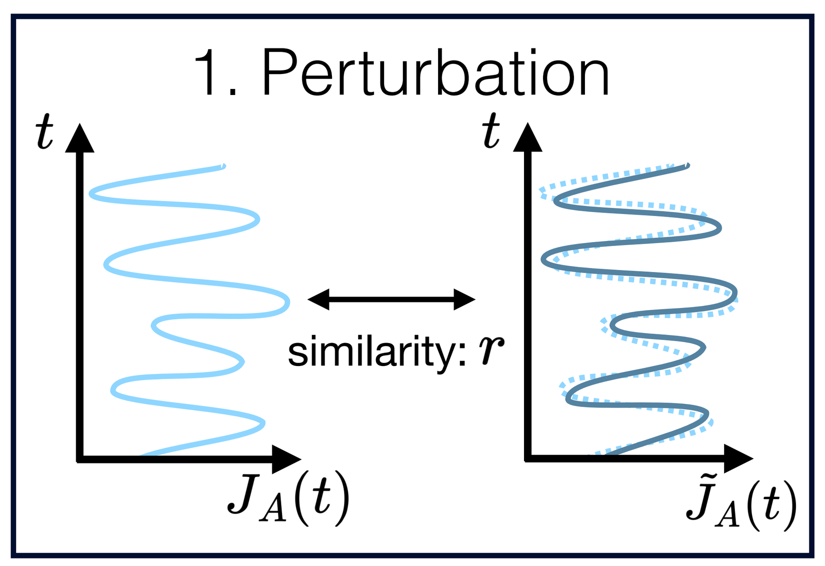
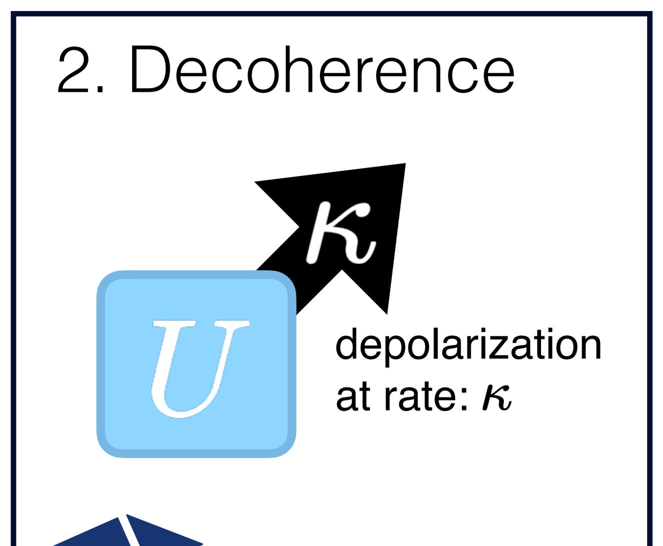
Measure of Scrambling

$$\mathrm{OTOC} = \langle [W, V(t)]^\dagger [W, V(t)] \rangle$$

Two Imperfections (tuned by r and κ)

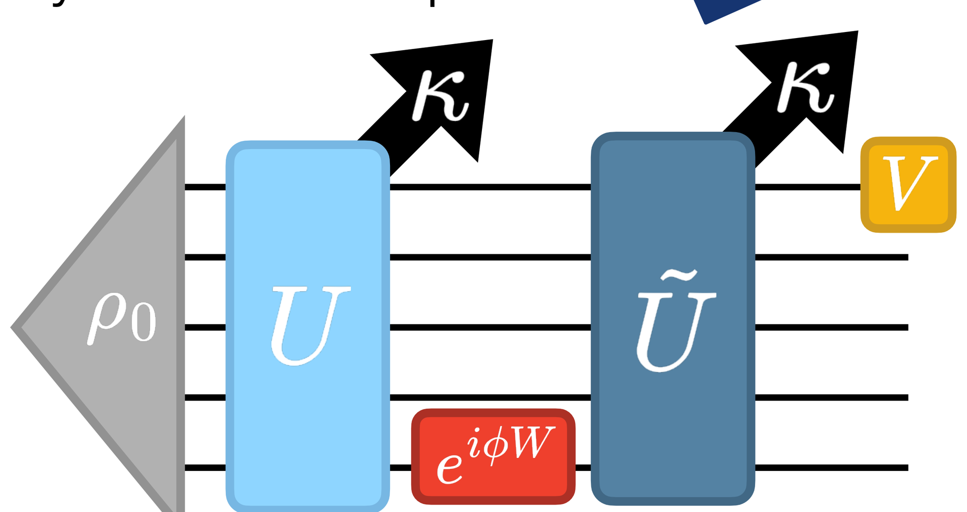




Ask me!

Protocol

Inspired by the NMR experiment



1. Prepare the initial state, $\rho_0 = \dfrac{I + \epsilon V}{2^N}$
2. Evolve forward, $\rho \to U\rho U^\dagger$
3. Rotate, $\rho \to e^{i\phi W}\rho e^{-i\phi W}$
4. Evolve backward with perturbation, $\rho \to \tilde{U}^\dagger \rho \tilde{U}$
5. Measure V

$$\mathrm{tr}(\rho_f V) = \frac{\epsilon}{2^N}\mathrm{tr}\left(V\tilde{U}^\dagger e^{i\phi W} U V U^\dagger e^{-i\phi W}\tilde{U}\right)$$
$$= \epsilon\langle V\tilde{U}^\dagger e^{i\phi W} U V U^\dagger e^{-i\phi W}\tilde{U}\rangle$$
$$= \epsilon f_{\mathrm{isolated}}(\phi, U, \tilde{U})$$

*Can be easily promoted to channel with noise rate κ

$$\mathrm{ROTOC} \equiv \frac{\partial_\phi^2 f}{f}\Big|_{\phi=0} = \frac{\langle [W, \tilde{V}(t)]^\dagger [W, V(t)]\rangle_\infty}{\langle \tilde{V}(t) V(t)\rangle_\infty}$$

Main Results

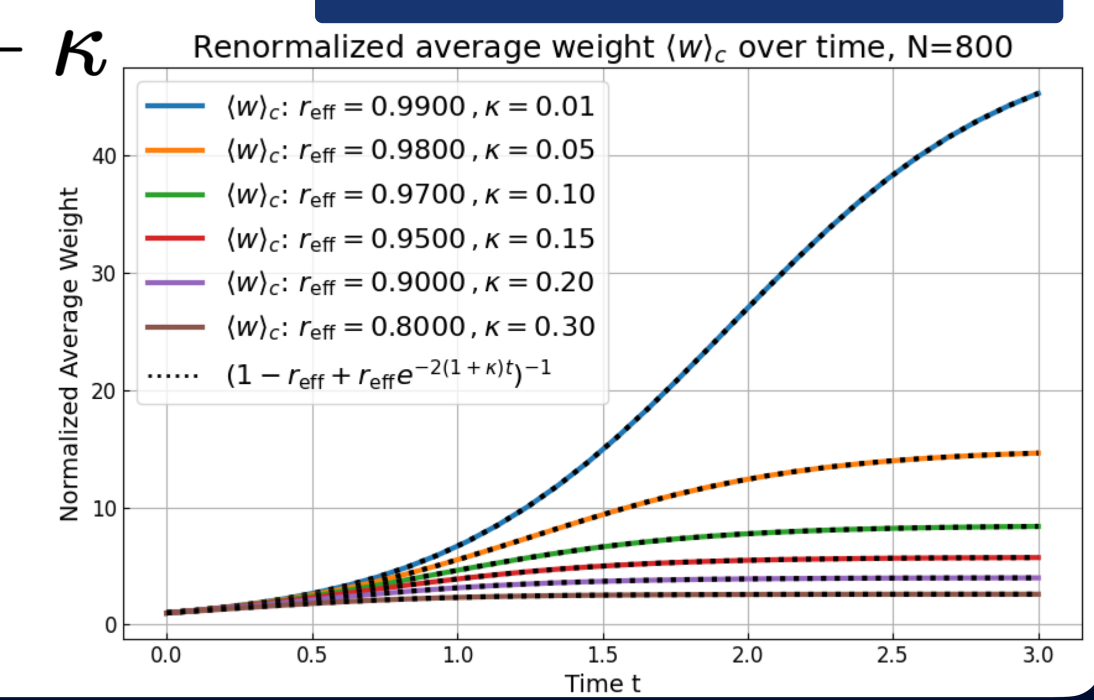
In dilute limit $N \to \infty$, and Pauli string observable with weight w_0

$$\mathrm{ROTOC} = \frac{3}{8N}\,\frac{w_0}{1 - r_{\mathrm{eff}} + r_{\mathrm{eff}}e^{-2(1+\kappa)t}}, \quad r_{\mathrm{eff}} \equiv \frac{r}{1+\kappa}$$

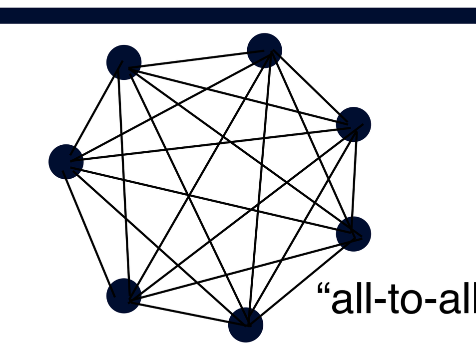
Early time **scrambling exponent is unaffected** up to rescaling of time

$$\mathrm{ROTOC}_{t\to 0} = \frac{w_0 e^{2(1+\kappa)t}}{1 - r_{\mathrm{eff}}} + \dots$$

Future: analyze advantage of OTOC with imperfections



Solvable Model - Discrete Brownian Circuit



Pauli species

$$H(t) = \sum_{i<j,\alpha\beta}^{N} J_{ij}^{\alpha\beta}(t)\sigma_i^\alpha\sigma_j^\beta = \sum_A J_A O_A$$

site — Gaussian random variable

$$\mathbb{E}[J_A(t)\tilde{J}_{A'}(t')] = r\mathbb{E}[J_A(t)J_{A'}(t')]$$

Expand operators in Pauli basis $V(t) = \sum_P c_P(t)P$

Ensemble average the observables

$$\mathrm{ROTOC} = \frac{\sum_{P|\{P,W\}=0} 4\mathbb{E}[c_P\tilde{c}_P]}{\sum_P \mathbb{E}[c_P\tilde{c}_P]}$$

Statistical symmetry —> consider in weight space

$$b_w \equiv \sum_{P|\mathrm{wt}(P)=w}\mathbb{E}[\tilde{c}_P c_P] \xrightarrow{\text{renormalize}} c_w(t) \equiv \frac{b_w(t)}{\sum_{w'} b_{w'}(t)}$$

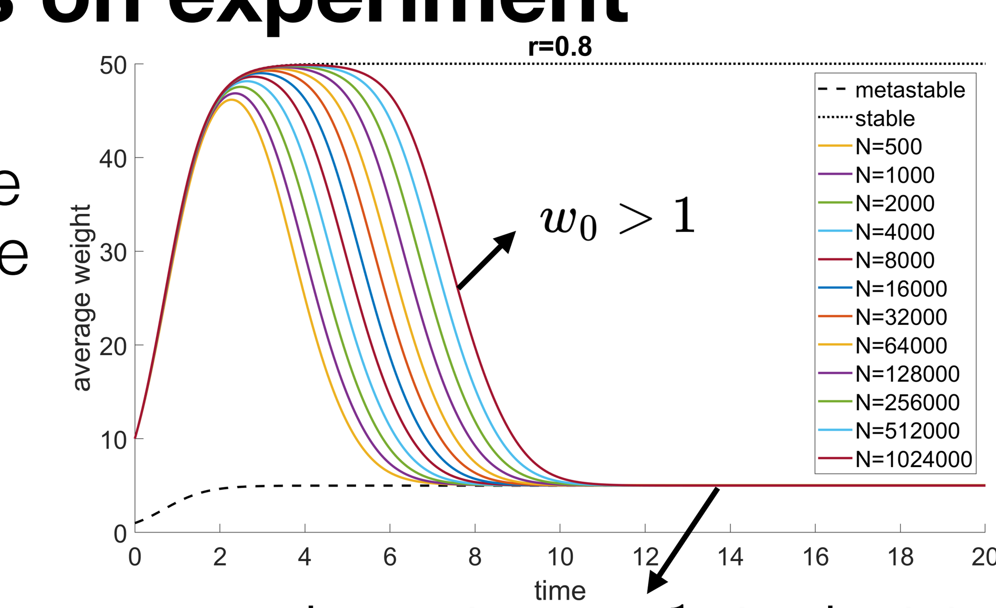
probability distribution

$$\mathrm{ROTOC} = \frac{8\langle w\rangle_c}{3N}$$

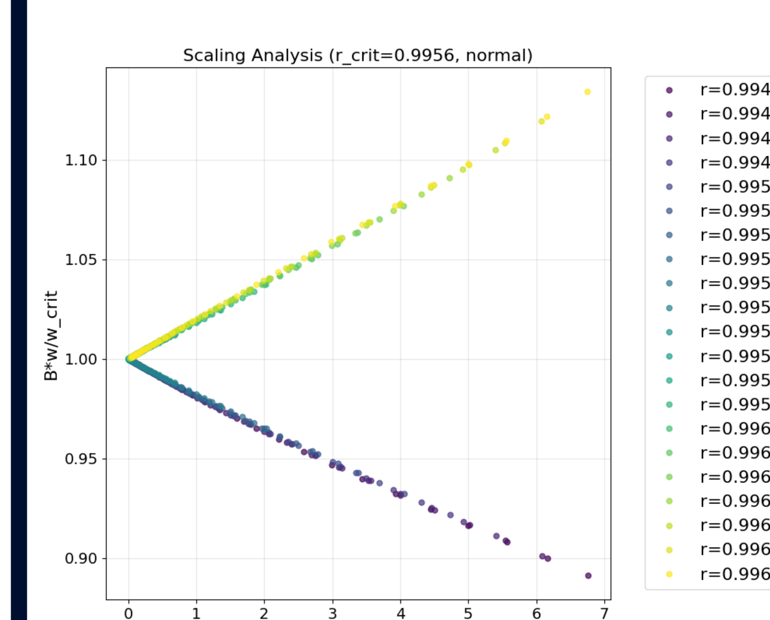
Metastable states and comments on experiment

For $w_0 > 1$, metastable states are observed with lifetime (dilute limit)

$$\tau \sim \ln\frac{N}{w_0}$$



decays to $w_0 = 1$ steady state



Original experiments:

Phase transition w.r.t r_{crit}

Our model:

Any r_{crit} will result in a scaling collapse on the left

Ask me!

Rapid crossover at $p_* \approx .028$ comparable to $p_c = .027$

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